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### How to Use This Addenda

Make sure you're ready to teach by noting the **Necessary Materials and Pre-Lesson Prep** you will need to gather or complete prior to the lesson

Find high-leverage instructional moves in the **Lesson Look Fors**. This is what leaders should see when observing your instruction

Note how your lesson objective ties to your state **Standards**

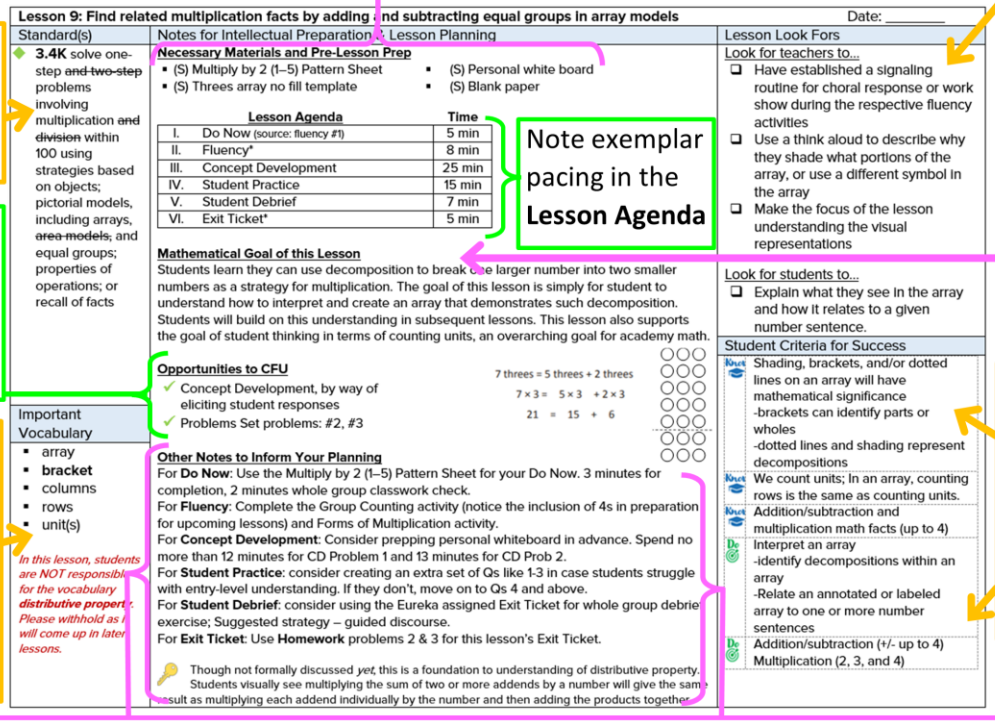
Note exemplar pacing in the **Lesson Agenda**

Use the **Mathematical Goal of the Lesson** to keep you focused on the appropriate student outcome

Plan purposeful questioning and responses using **Opportunities to CFU**

Plan instruction around what students need to Know & Do to be successful on the Exit Ticket using the identified **Student Criteria for Success**

Plan to stress **Important Vocabulary** in the lesson. New vocab for the unit is indicated in bold



**Lesson 9: Find related multiplication facts by adding and subtracting equal groups in array models** Date: _____

**Standard(s)**

**3.4K** solve one-step ~~and two-step~~ problems involving multiplication ~~and division~~ within 100 using strategies based on objects; pictorial models, including arrays, ~~area models~~, and equal groups; properties of operations; or recall of facts

**Notes for Intellectual Preparation & Lesson Planning**

**Necessary Materials and Pre-Lesson Prep**

- (S) Multiply by 2 (1–5) Pattern Sheet
- (S) Threes array no fill template
- (S) Personal white board
- (S) Blank paper

| Lesson Agenda | | Time |
|---|---|---|
| I. | Do Now (source: fluency #1) | 5 min |
| II. | Fluency* | 8 min |
| III. | Concept Development | 25 min |
| IV. | Student Practice | 15 min |
| V. | Student Debrief | 7 min |
| VI. | Exit Ticket* | 5 min |

**Mathematical Goal of this Lesson**

Students learn they can use decomposition to break one larger number into two smaller numbers as a strategy for multiplication. The goal of this lesson is simply for student to understand how to interpret and create an array that demonstrates such decomposition. Students will build on this understanding in subsequent lessons. This lesson also supports the goal of student thinking in terms of counting units, an overarching goal for academy math.

**Opportunities to CFU**

- ✓ Concept Development, by way of eliciting student responses
- ✓ Problems Set problems: #2, #3

7 threes = 5 threes + 2 threes
7 × 3 = 5 × 3 + 2 × 3
21 = 15 + 6

**Important Vocabulary**

- array
- **bracket**
- columns
- rows
- unit(s)

*In this lesson, students are NOT responsible for the vocabulary **distributive property**. Please withhold as it will come up in later lessons.*

**Other Notes to Inform Your Planning**

For **Do Now**: Use the Multiply by 2 (1–5) Pattern Sheet for your Do Now. 3 minutes for completion, 2 minutes whole group classwork check.
For **Fluency**: Complete the Group Counting activity (notice the inclusion of 4s in preparation for upcoming lessons) and Forms of Multiplication activity.
For **Concept Development**: Consider prepping personal whiteboard in advance. Spend no more than 12 minutes for CD Problem 1 and 13 minutes for CD Prob 2.
For **Student Practice**: consider creating an extra set of Qs like 1-3 in case students struggle with entry-level understanding. If they don't, move on to Qs 4 and above.
For **Student Debrief**: consider using the Eureka assigned Exit Ticket for whole group debrief exercise; Suggested strategy – guided discourse.
For **Exit Ticket**: Use **Homework** problems 2 & 3 for this lesson's Exit Ticket.

Though not formally discussed *yet*, this is a foundation to understanding of distributive property. Students visually see multiplying the sum of two or more addends by a number will give the same result as multiplying each addend individually by the number and then adding the products together.

**Lesson Look Fors**

Look for teachers to...

- ❑ Have established a signaling routine for choral response or work show during the respective fluency activities
- ❑ Use a think aloud to describe why they shade what portions of the array, or use a different symbol in the array
- ❑ Make the focus of the lesson understanding the visual representations

Look for students to...

- ❑ Explain what they see in the array and how it relates to a given number sentence.

**Student Criteria for Success**

Know: Shading, brackets, and/or dotted lines on an array will have mathematical significance
-brackets can identify parts or wholes
-dotted lines and shading represent decompositions

Know: We count units; In an array, counting rows is the same as counting units.

Know: Addition/subtraction and multiplication math facts (up to 4)

Do: Interpret an array
-identify decompositions within an array
-Relate an annotated or labeled array to one or more number sentences

Do: Addition/subtraction (+/- up to 4)
Multiplication (2, 3, and 4)

Find recommended lesson modifications, content knowledge boosters, and/or high-leverage instructional moves that may not be in your Teacher Edition located in **Other Notes to Inform Your Planning**

## UNIT SYNOPSIS

In early grades, students investigated the properties of circles and in Grade 7, students defined pi, $\pi$, as the ratio of the circumference to the diameter of a circle. Since students have already had Algebra 1, they now are introduced to the equation of a circle. Then, they learn circle parts beyond "radius" and "diameter," including "chord" and "tangent" and their properties. Students also learn about angles created by circle parts, including inscribed angles and central angles. Toward the end of the unit, students revisit circumference and area so that they may develop an understanding of area of a sector and arc length.

## CONTENT STANDARDS

Below are the standards addressed in this unit.

| Readiness Standards | Supporting Standards |
|---|---|
| **G.11(B)** determine the area of ~~composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or~~ sectors of circles to solve problems using appropriate units of measure | **G.12(A)** apply theorems about circles, including relationships among angles, radii, chords, tangents, and secants, to solve non-contextual problems<br>**G.12(B)** apply the proportional relationship between the measure of an arc length of a circle and the circumference of the circle to solve problems<br>**G.12(E)** show that the equation of a circle with center at the origin and radius is $x^2 + y^2 = r^2$ and determine the equation for a graph of a circle with radius $r$ and center $(h, k)$, $(x - h)^2 + (y - k)^2 = r^2$; |

| Focus on Disciplinary Literacy | |
|---|---|
| | Mathematical Process Standard (F) – analyze mathematical relationships to connect and communicate mathematical ideas |
| | Mathematical Process Standard (G) – display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication |

## LEARNING SUPPORTS BY LESSON

| There is a checkmark for the math support if the lesson | Lessons → Math Supports | L1 | L2 | L3 | L4 | L5 | L6 | L7 | L8 | L9 | L10 | L11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| makes a connection to prior content or from a previous unit or academic year | Access Prior Knowledge | ✓ | ✓ | ✓ | | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| uses familiar contexts or experiences to make the learning relevant to students | Real-World Connections | ✓ | ✓ | | ✓ | ✓ | | | ✓ | ✓ | ✓ | |
| makes use of graphic organizers | Graphic Organizers | ✓ | | ✓ | | | | | | | | |
| includes tools like rulers, protractors, patty paper, algebra tiles, etc. | Tools or Manipulatives | ✓ | | ✓ | ✓ | ✓ | | | ✓ | | ✓ | |
| incorporates tables, reference charts, displays, pictures, models, or color-coding | Visual Aids | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| includes definitions, examples vs. nonexamples, cognates, etc. | Vocabulary Supports | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | ✓ | ✓ | ✓ | |
| includes strategies that support language development | | | | | | | | | | | | |
| asks students to discuss with their partner to prepare for whole class discussion | - Turn and Talk | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| teacher facilitates a whole class discussion to debrief key learnings | - Guided Discussion | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| asks students to think independently, test their idea with a partner, and share whole group | - Think, Pair, Share | | | | | | | | | | | |
| includes sentence stems to support students with explanations | - Sentence Stems | | | | | | | | | | | |
| provides opportunities for students to work with a partner or a group | Peer Collaboration | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| uses mnemonics such as SohCahToa | Mnemonics | | | | | | | | | | | |
| includes websites or equipment that enhances the lesson | Technological Support | | | | | | ✓ | | | | | |
| content can be presented in different forms | | | | | | | | | | | | |
| uses hands-on tools or manipulatives to represent the math | - Concrete | | | | ✓ | | | | ✓ | | ✓ | |
| uses drawings to represent the math | - Pictorial | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| uses numbers and number sentences to represent the math | - Abstract | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

# The EFFL Model

## Experience First, Formalize Later (EFFL) Model

### Opening

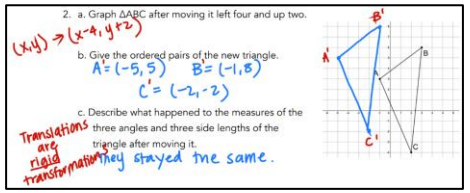
For every new lesson, the teacher begins by making the goals of the lesson crystal clear. The teacher does more than simply read the objective to the class. They make connections to previous learning, share how this learning fits into a bigger picture, or explain why this learning is important for future learning.

### Activity / Interaction With New Material (INM)

For this part of the lesson, students work in pairs or groups of four to experience new content through an activity. Students might be discussing a proposed scenario, working with other groups, or doing a simulation. The student activity is designed for students to be able to do without the help of the teacher. Of course, the teacher is watching and listening in to conversations in order to formatively assess student understanding. The teacher provides questions, cues, and prompts (not answers!) to help push groups forward when they are stuck or have made a mistake. As students begin to finish the activity, the teacher identifies students to write their work on the board. Most often, the teacher selects student work that will easily allow them to connect the experience to formal learning. Students write their work on the whiteboard in a single-color marker.

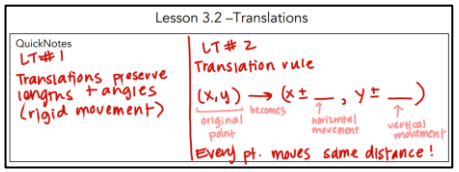
### Debrief Activity

Once students have recorded their responses in their workbook (see blue writing to the right), the teacher calls the whole group back together for a debrief. It is in this discussion that the teacher will help students formalize the learning. The teacher connects the student activity experience to new vocabulary, definitions, formulas, and algorithms. The formal learning is attached specifically to the experiences of the activity so that students can enhance their constructed understanding of the new content. The teacher writes all of the formal learning in a different color in the margins of the activity **(see red writing to the right).** The students add these ideas in the margins on their activity page and often think of this as the formal "notes" of the lesson. In all of the answer keys we provide on Math Medic, the teacher formal learning points are provided in the margins in a **different color.**




### QuickNotes

In this part of the lesson, the teacher uses the whole experience of the activity and the formalization in the debrief to summarize the learning from the lesson. Notice that we use the box to constrain the amount of formal "notes" that the teacher can provide.




### Student Practice

Now that students have arrived at some new learning, they need to be able to apply it in new contexts. Most often we have students complete these questions in pairs and occasionally we select one question to use as an exit ticket. If we have time, we have students write solutions on the whiteboard.

### Extra Practice

We typically give students around 3-5 "Extra Practice" problems for each lesson. We choose problems that are closely aligned with the Learning Objectives of the lesson. It is our belief that "less is more" here. We would rather students spend their Extra Practice time thinking deeply about just a few problems, rather than surface level thinking on many problems. When possible, we provide the answers at the bottom of the page, so they can immediately assess their understanding.

Slightly modified version of: https://www.calc-medic.com/post/experience-first-formalize-later#:~:text=%E2%80%9CExperience%20First%2C%20Formalize%20Later%E2%80%9D,at%20formal%20definitions%20and%20formulas.

## Before You EFFL!

Here are helpful resources that you guide you in the right direction before your first EFFL lesson!

**Why Should We EFFL?**

The article advocates for the Experience First, Formalize Later (EFFL) teaching model, emphasizing its effectiveness in fostering deep understanding and flexible thinking in students. The author compares traditional teaching to a game of "Simon Says," where students merely mimic instructions without grasping underlying concepts. In contrast, EFFL encourages students to engage actively with problems, enhancing their ability to understand and apply calculus concepts creatively.

**Tips for Lesson Planning**

The article offers practical advice for effective lesson planning beyond the exhaustive and overly detailed approaches often emphasized during teacher training. It underscores the importance of thoughtful preparation but rejects the notion that teachers need to script every minute or detail of a class session.

**Making the Most of Your EFFL Lesson Debrief**

The article discusses the significance of the debriefing phase in the Experience First, Formalize Later (EFFL) lesson model, emphasizing its role in reinforcing learning and highlighting student contributions. The debrief session is seen as crucial for integrating academic vocabulary, emphasizing key lesson understandings, and valuing students' mathematical insights.

## While You EFFL!

While each lesson may be unique in context and skills, all lessons benefit from the following practices:

**Teacher Look Fors:**

- ❑ Utilizing the Do Now to spark students' interest in the Activity.
- ❑ Use questioning to promote small group discussion and exploration, guided by monitoring questions.
- ❑ Connects Experience First to formal concepts using a colored pencil/pen to take notes along the margin during the Debrief.
- ❑ Facilitates whole-class discussions for students to reflect, share insights, and provides feedback that reinforces key concepts.
- ❑ Tracks time to adapt lesson pacing and support based on student response and engagement.

**Students Look Fors:**

- ❑ In the Activity, students engage in group work and discourse.
- ❑ Exploring the activity, testing hypotheses and approaches (trial & error).
- ❑ Take notes on key ideas and concepts using different colored pencil/pen to take notes along the margin.
- ❑ Share thoughts and ideas that demonstrate their approach to their work.

**Other considerations**

- During the **Experience First** phase, if most of your students seem stuck or disengaged, take a moment to pause, reset, and provide clear instructions. Some problems of the Activity are more suitable to do a whole-class discussion as a means to save some instructional time for Student Practice or the Exit Ticket. You are encouraged to adapt the EFFL (Experience First, Formalize Later) process to meet your students' needs while maintaining a focus on student-centered instruction.

## ROADMAP

| AT A GLANCE: Unit 8 – Circles | | | |
|---|---|---|---|
| Day | Date | Lesson | Lesson Title |
| There are 5 flexible Success Days that you can use anywhere in the unit.<br>• Consider using 1 day to facilitate Lesson 7, a Mid-Unit review that addresses content from Lessons 1 - 6.<br>• Consider using 1 day to administer Topic Quiz A between Lessons 7 and 8, and another day to administer Topic Quiz B after Lesson 10.<br>• Consider using 1 day to review the day before UE8. For this review, consider using the provided review lesson, Lesson 8.11.<br>• If you don't need to use all five flexible Success Days, consider saving one or more for future units. | | | |
| 1 | | 1 | Equation of a Circle |
| 2 | | 2 | Circle Vocabulary |
| 3 | | 3 | Tangent Lines |
| 4 | | 4 | Chords and Arcs |
| 5 | | 5 | Perpendicular Bisectors of Chords |
| 6 | | 6 | Inscribed Angles and Quadrilaterals |
| 7 | | 7 | Mid-Unit Review A (Lessons 1 – 6) |
| 8 | | | Unit 8 Topic Quiz A \| Success Day |
| 9 | | 8 | Area and Circumference of a Circle |
| 10 | | 9 | Area of a Sector |
| 11 | | 10 | Arc Length |
| 12 | | | Unit 8 Topic Quiz B \| Success Day |
| 13 | | CR | Cumulative Review Lesson 8.11 \| Success Day |
| 14 | | | Unit 8 Exam |
| 15 | | | Success Day |

Lesson 1: Equation of a Circle Date: _____

## Standard(s)

◆ **G.12(E)** show that the equation of a circle with center at the origin and radius is $x^2 + y^2 = r^2$ and determine the equation for a graph of a circle with radius $r$ and center $(h, k)$, $(x - h)^2 + (y - k)^2 = r^2$

## Important Vocabulary

- **center**
- **circle**
- **radius**

## Notes for Intellectual Preparation & Lesson Planning

**Necessary Materials and Pre-Lesson Prep**

- Lesson 8.1 Student Workbook pages
- String (enough for each student to cut about 3 inches, with extra for mistakes)
- Class set of red pens, scissors, and rulers

Lesson Structure:

- Do Now (7 min)
- INM (21 min)
- Debrief (2 min)
- Student Practice (15 min)
- Exit Ticket (10 min)

**Mathematical Goal of this Lesson**

By the end of this lesson, students should be able to write the equation for a circle or graph a circle given an equation. To do this, students must be able to understand what each part of the equation of a circle represents. This lesson builds on students' understanding of the Pythagorean Theorem to make sense of why the circle formula works.

**Opportunities to CFU**

✓ INM: 2, 3 4c, 12

✓ Student Practice: 1, 2, 3, 4

**Other Notes to Inform Your Planning**

For the **Do Now:** The Do Now sets up the INM and cannot be skipped or replaced.

On **scaffolding**: If you anticipate students will struggle to work through the Do Now, consider providing pre-cut string that represents the 5 mile distance and give an example of how to use it when debriefing the Do Now, since they'll need to be able to use string or ruler to do this for later questions.

**Focus on Disciplinary Literacy**

Debrief

On **pacing**: If you worry about running out of time during the INM, be sure to look at the pacing suggestions provided in the pink boxes on pages 7 and 9.

## Lesson Look Fors

Look for teachers to…

- ❑ circulate and monitor as students are working on the INM, pre-selecting which students to call for the debrief.
- ❑ when debriefing #s 8-9, emphasize the connection between the Pythagorean Theorem and the equation of a circle, and "stamp" the meaning of "center" and "radius."

Look for students to…

- ❑ use string and/or a ruler and the "2 mile" key to work through the first page of the lesson (Do Now and INM).
- ❑ apply the Pythagorean Theorem (or Distance Formula) to work through the second page of the INM.

## Student Know/Do Chart

Do: Students can write the equation of a circle that is graphed on a coordinate plane.

Know: The equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$

Know: In the equation of a circle, $(x - h)$ and $(y - k)$ represent net distances, which is why the center is $(h, k)$, and not $(-h, -k)$.

Lesson 2: Circle Vocabulary Date: _____

## Standard(s)

- **G.12(A)** apply theorems about circles, including relationships among angles, radii, ~~chords, tangents, and secants,~~ to solve non-contextual problems

## Important Vocabulary

- **arc**
- **central angle**
- **chord**
- circle
- **diameter**
- radius
- **major arc**
- **minor arc**
- **semicircle**

## Notes for Intellectual Preparation & Lesson Planning

**Necessary Materials and Pre-Lesson Prep**

- Lesson 8.2 Student Workbook pages
- Class set of red pens

Lesson Structure:

- Do Now (7 min)
- INM (16 min)
- Debrief (5 min)
- Student Practice (17 min)
- Exit Ticket (10 min)

**Mathematical Goal of this Lesson**

By the end of this lesson, students should be able to define and identify and/or distinguish between a circle's major and minor arcs, chords, tangent lines, radius, and diameter. They should also develop the understanding that the measure of an arc is the measure of its corresponding central angle. The arc measure and the arc length are NOT the same thing – students will learn about arc length near the end of the unit.

**Opportunities to CFU**

✓ INM: 2, 4, 7

✓ Student Practice: 1, 2, 3, 4

**Other Notes to Inform Your Planning**

For the **Do Now:** The Do Now sets up the INM and cannot be skipped or replaced.

On **scaffolding**: If students struggle to understand #2, try asking, "How far is George away from the cart? How do you know?" If that doesn't work, try, "Think of the Ferris wheel as a circle. What do you call the distance from the center to the edge of the circle? Is it the same length every time?" (You are trying to get them to see that a circle's radii are congruent.) For #4, some students might not know that a full circle is 360°. If they don't already know this, please tell them! If they don't know how to begin INM #s 4-6, help them see that the Ferris wheel is kind of like a pizza where each slide is exactly the same size (so one central angle has to be $\frac{360°}{12} = 30°$.

**Focus on Disciplinary Literacy**

Debrief

## Lesson Look Fors

Look for teachers to…

- ❑ avoid giving students definitions to copy before engaging in the Do Now / INM.
- ❑ when trying to help struggling students "get unstuck," try using the scaffolding questions provided below to the left. Rather than give away information, questions should build on what students learned about "radius" from the previous lesson.

Look for students to…

- ❑ mark up the Ferris wheel diagram as much as they need to.
- ❑ recall what they already learned about circles in 7th grade and apply it whenever they can.

## Student Know/Do Chart

| | |
|---|---|
| Do | Students can identify and name a major arc and minor arc given a diagram of a circle. |
| Do | Students can determine an arc measure given a diagram of a circle with central angle measures. |
| Know | Minor arcs are named with two letters, like so: $\overset{\frown}{HJ}$. Major arcs are named with three letters so that it's easy to tell which direction around the circle you're going, like so: $\overset{\frown}{HJG}$. |
| Know | The measure of an arc is the measure of its corresponding central angle. |
| Know | The central angles of a circle sum up to a total of 360°. |

Lesson 3: Tangent Lines Date: _____

## Standard(s)

- **G.12(A)** apply theorems about circles, including relationships among angles, radii, ~~chords,~~ tangents, ~~and secants~~, to solve non-contextual problems

## Important Vocabulary

- **tangent**
- **point of tangency**

## Notes for Intellectual Preparation & Lesson Planning

**Necessary Materials and Pre-Lesson Prep**

- Lesson 8.3 Student Workbook pages
- Class set of red pens, rulers, and protractors

**Lesson Structure:**

- Do Now (7 min)
- INM (18 min)
- Debrief (7 min)
- Student Practice (13 min)
- Exit Ticket (10 min)

**Mathematical Goal of this Lesson**

By the end of this lesson, students should be able to apply properties of tangents to find missing values.

**Opportunities to CFU**

- ✓ INM: 4, 5
- ✓ Student Practice: 1, 2, 3, 4

**Other Notes to Inform Your Planning**

**Before you teach the lesson:** Complete INM #s 1-5 on your own and write out all the measurements. You'll need this so that students can play the game in INM #6.

For the **Do Now:** The Do Now is not part of the INM. It can be skipped or replaced, although it does serve to spiral in previous knowledge.

On **scaffolding**: If you anticipate students will not know how to use a protractor (required for #5), consider modeling how to when going over the Do Now. When it's time for the game in INM #6, have project the table in INM #6 and write in the information as you give it. Write down the information students give you in response so everyone can see where they are in the process.

**Focus on Disciplinary Literacy**

DEBRIEF

## Lesson Look Fors

Look for teachers to…

- ❑ Circulate and monitor while students are engaging in INM #s 1-5.
- ❑ give clear directions for the game in INM #6, and have their own measurements in hand BEFORE the game starts.
- ❑ When debriefing, use what students noticed in INM #s 1 – 5 (tangents are perpendicular to the radius at the point of tangency) and INM #6 (other tangent properties) to formalize the tangent properties shown in the QuickNotes.

Look for students to…

- ❑ carefully follow directions in INM #s 1 – 5.
- ❑ realize, upon comparing their work with their colleagues, that they each created two right angles!
- ❑ try their best and have fun playing "Gimme 10" in INM #6 with their teacher.

## Student Know/Do Chart

Students can apply properties of tangents to find missing values.

A tangent line is perpendicular to a circle's radius drawn at the point of tangency.

If two segments are tangent to the same circle and they share a common endpoint, then those two segments are congruent.

Lesson 4: Chords and Arcs | Date: _____

## Standard(s)

- **G.12(A)** apply theorems about circles, including relationships among angles, radii, chords, tangents, ~~and secants~~, to solve non-contextual problems

## Important Vocabulary

- arc
- chord

## Notes for Intellectual Preparation & Lesson Planning

**Necessary Materials and Pre-Lesson Prep**

- Lesson 8.4 Student Workbook pages
- Extra copies of the cookie page (for students who choose to cut)
- Class set of red pens, protractors, rulers, string, and scissors

Lesson Structure:

- Do Now (7 min)
- INM (15 min)
- Debrief (7 min)
- Student Practice (16 min)
- Exit Ticket (10 min)

**Mathematical Goal of this Lesson**

By the end of this lesson, students should be able to apply properties of chords to find missing values. The big takeaway of today's lesson is that when two chords in a circle are congruent, their corresponding arc measures are congruent and the central angles made by the chords are congruent.

**Opportunities to CFU**

- ✓ INM: 2a, 2b, 2c
- ✓ Student Practice: 1, 2, 3, 4

**Other Notes to Inform Your Planning**

For the **Do Now:** The Do Now sets up the INM and cannot be skipped or replaced.

On **scaffolding**: As the TE mentions on p46, this lesson honors diverse student contributions and highlights multiple mathematical abilities. This is because students are allowed to use any tools you provide them to answer the INM questions.

**Focus on Disciplinary Literacy**

INM

## Lesson Look Fors

Look for teachers to…

- ❑ circulate and monitor as students engage with the INM, preselecting students to share responses during the Debrief and encouraging students as they try different methods of measuring.
- ❑ when debriefing, have various students explain their measuring tool of choice and their method for determining if the two pieces were even. Keep asking, "Did anybody prove it in a different way?" (see TE p35)

Look for students to…

- ❑ try a variety of methods of measurement; perhaps cut pictures and superimpose pieces onto each other, measure lengths with a ruler, or use string to find arc length or perimeters of regions.
- ❑ for each Method (part a, b, and c), justify their choice of measurement tool (e.g. "I used a protractor to check the angle measure)

## Student Know/Do Chart

Know — If two chords in a circle are congruent, then their corresponding arc measures are congruent and the central angles made by the chords are congruent.

Know — If two chords in a circle are congruent, there is no guarantee that the central angle created by the endpoint of one chord and the endpoint of the other chord is congruent to the central angle created by the other endpoint of the original chord and the other endpoint of the other chord.

Lesson 5: Perpendicular Bisectors of Chords Date: _____

## Standard(s)

- **G.12(A)** apply theorems about circles, including relationships among angles, radii, chords, tangents, ~~and secants~~, to solve non-contextual problems

## Important Vocabulary

- arc
- chord
- **perpendicular bisector**

## Notes for Intellectual Preparation & Lesson Planning

**Necessary Materials and Pre-Lesson Prep**

- Lesson 8.5 Student Workbook pages
- Class set of red pens, rulers, protractors

Lesson Structure:

- Do Now (7 min)
- INM (15 min)
- Debrief (7 min)
- Student Practice (16 min)
- Exit Ticket (10 min)

**Mathematical Goal of this Lesson**

By the end of this lesson, students should be able to apply properties of perpendicular bisectors of chords to find missing values.

**Opportunities to CFU**

- ✓ INM: 1, 2, 3, 4
- ✓ Student Practice: 1, 2

**Other Notes to Inform Your Planning**

For the **Do Now:** The Do Now sets up the INM and cannot be skipped or replaced. The TE exemplar for the Do Now on p56 shows more than student will do during the Do Now. During the Do Now, the students should only be answering 1a and b, but they will not yet have drawn a diameter or chord yet. The other sketches will be made by students as they progress through the INM.

On **scaffolding**: Students need to understand #2 before moving on. If they do not, revisit the information in the box about the signal that travels 30 meters in all directions. Once they understand #2, help them identify any point on the circle for A and use their ruler to draw a line from A to another point on the circle, B. If they're stuck on #4, have them use their ruler to identify the midpoint and label it M.

**Focus on Disciplinary Literacy**

DEBRIEF

## Lesson Look Fors

Look for teachers to…

- ❑ avoid telling students that the perpendicular bisector of a chord passes through the center of a circle BEFORE the debrief.
- ❑ circulate and monitor, preselecting students to share their ideas during the Debrief. If students get too stuck on #s 2-5, re-read or re-state the directions and support them as they attempt to follow the directions.

Look for students to…

- ❑ carefully read the INM questions so they can follow the directions given that will allow them to draw a chord and its perpendicular bisector.
- ❑ by the end of the Debrief (ideally, by the end of the INM), realize that a chord's perpendicular bisector passes through the center of the circle.

## Student Know/Do Chart

Students can identify from a given diagram that a diameter is perpendicular to a chord.

Students can set up and solve an equation.

If a diameter is perpendicular to a chord, the diameter bisects the chord.

Lesson 6: Inscribed Angles and Quadrilaterals Date: _____

## Standard(s)

- **G.12(A)** apply theorems about circles, including relationships among angles, radii, chords, tangents, ~~and secants,~~ to solve non-contextual problems

## Important Vocabulary

- inscribed angle
- inscribed quadrilateral
- intercepted arc

## Notes for Intellectual Preparation & Lesson Planning

**Necessary Materials and Pre-Lesson Prep**

- Lesson 8.6 Student Workbook pages
- Class set of red pens
- Class set of laptops with GeoGebra access

**Lesson Structure:**

- Do Now (7 min)
- INM (23 min)
- Debrief (3 min)
- Student Practice (12 min)
- Exit Ticket (10 min)

**Mathematical Goal of this Lesson**

By the end of this lesson, students should be able to apply properties of inscribed angles and inscribed quadrilaterals to find missing values. Students get to see these properties in action during two different GeoGebra investigations!

**Opportunities to CFU**

- ✓ INM: 2, 5, 10, 11
- ✓ Student Practice: 1, 2, 3

**Other Notes to Inform Your Planning**

For the **Do Now:** The Do Now sets up the INM and cannot be skipped or replaced. The Do Now includes time to get to GeoGebra.

On **scaffolding**: GeoGebra is a fantastic way for students to see and be able to manipulate inscribed and central angles. If you anticipate the majority of students will struggle with GeoGebra, have it ready on your projector and guide students through the first few steps. However, it is worth giving students the opportunity to try it first, since the directions are clearly written out on the workbook pages.

**Focus on Disciplinary Literacy**

DEBRIEF

## Lesson Look Fors

Look for teachers to…

- ❑ circulate and monitor as students engage in the GeoGebra explorations, preselecting groups to share what they notice during the Debrief.
- ❑ ensure students don't stay on Investigation #1 too long – after about 10 minutes after the Do Now, tell students they should almost be moving onto Investigation #2 if they aren't already there.

Look for students to…

- ❑ carefully follow the GeoGebra directions provided in the INM.
- ❑ compare their own findings with other group members, especially in #s 5 and 9.

## Student Know/Do Chart

| | |
|---|---|
| Do | Students can identify the measure of an inscribed angle given the measure of a central angle that has the same endpoints on the circle. |
| Do | Students can solve for arc measures given one arc measure and a central angle measure. |
| Know | An angle that has a vertex on a circle and rays that are cords of the circle is half the measure of an central angle with the same endpoints. |
| Know | The measure of an arc is the same as its corresponding central angle. |
| Know | The sum total of a circle's non-overlapping arc measures is 360°. |

Lesson 7: Mid-Unit Review A (Lessons 1-6) – **CASTLE ATTACK!** Date: _____

## Standard(s)

- **G.12(A)** apply theorems about circles, including relationships among angles, radii, chords, tangents, ~~and secants,~~ to solve non-contextual problems
- **G.12(E)** show that the equation of a circle with center at the origin and radius is $x^2 + y^2 = r^2$ and determine the equation for a graph of a circle with radius $r$ and center $(h, k)$, $(x - h)^2 + (y - k)^2 = r^2$

## Important Vocabulary

All vocabulary from Lessons 1 – 6.

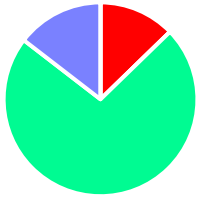
## Notes for Intellectual Preparation & Lesson Planning

**Necessary Materials and Pre-Lesson Prep**

- Unit 8.7 Student Workbook pages
- Whiteboard space for students

Lesson Structure:

- Do Now (7 min)
- INM (0 min)
- Castle Attack (40 min)
- Debrief (8 min)
- Exit Ticket (0 min)



**Mathematical Goal of this Lesson**

This lesson provides a review of Lesson 1-6 and gives students the opportunity to work together.

**Opportunities to CFU**

- ✓ when giving Castle Attack directions (before you release students to participate in the activity)
- ✓ during the Debrief when you go over most-missed questions

**Other Notes to Inform Your Planning**

For the **Do Now:** The provided Do Now is helpful for activating prior knowledge, but it is not married to the INM and can be replaced with a spiraled review question of your choice.

For **Castle Attack**: Please carefully read the directions in the pink box on TE p81.

For **Debrief:** Go over questions students struggled with the most.

For the **Exit Ticket**: No SP is provided because this review lesson is essentially a LOT of student practice. No ET is provided because the next day should be a Topic Quiz day that will give you a great data point.

**Focus on Disciplinary Literacy**

Castle Attack

## Lesson Look Fors

Look for teachers to…

- ❑ give clear directions for "Castle Attack" so the activity can flow smoothly.
- ❑ ensure they have space on their desk to keep printed and cut problems organized

Look for students to…

- ❑ talk with their partners about the math problem they're working on at the time.
- ❑ persevere through challenge. If neither partner knows how to solve a problem, that is okay! They can try a different problem and come back to it later. If they're totally stuck, they can ask for help.

## Student Know/Do Chart

Students can apply all "Dos" from Lessons 1-6.

Students know all "Knows" from Lessons 1-6.

Lesson 8: Area and Circumference of a Circle Date: _____

## Standard(s)

◆ **G.11(B)** determine the area of ~~composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of~~ circles to solve problems using appropriate units of measure

## Important Vocabulary

- area
- **circumference**
- radius
- **radii**

## Notes for Intellectual Preparation & Lesson Planning

**Necessary Materials and Pre-Lesson Prep**

- Unit 8.8 Student Workbook pages
- Class set of red pens, scissors, glue or tape, and string.
- Pizza printouts (TE pp 97-98)

**Lesson Structure:**

- Do Now (7 min)
- INM (22 min)
- Debrief (3 min)
- Student Practice (13 min)
- Exit Ticket (10 min)

**Mathematical Goal of this Lesson**

By the end of this lesson, students should be able to find the area, circumference, radius, and/or diameter of a circle and generate the formulas for area and circumference.

**Opportunities to CFU**

✓ INM: 1, 2, 3, 5

✓ Student Practice: 1, 2, 3, 4

**Other Notes to Inform Your Planning**

For the **Do Now:** The Do Now is not part of the INM. It can be skipped or replaced, although it serves to spiral in students' previous understanding of finding area and perimeter to set them up for finding area of a circle and circumference.

**About the INM**: The student responses in the Teacher Edition may vary from what is actually true when you measure the pizzas in your printed TE. This is because the size of the images may have been altered when Office Depot printed them, so please ensure that when internalizing your lesson, you measure the pizzas in your TE to ensure that you have the measurements that accurately reflect the size of the images students will work with.

**Focus on Disciplinary Literacy**

DEBRIEF

## Lesson Look Fors

Look for teachers to…

- ❑ circulate and monitor, preselecting students who will share what they noticed during the debrief.
- ❑ build on what students have already learned about similarity

Look for students to…

- ❑ use string to figure out how many string lengths it takes to get halfway around the pizza and all the way around the pizza
- ❑ notice that it takes about 6 radii to wrap around the entire circle.
- ❑ make the connection between $A = bh$ and $A = \pi r^2$ after seeing how the slices of their pizza formed a pseudo-parallelogram

## Student Know/Do Chart

Do — Students can find the circumference of a circle give its area.

Do — Students can operate with $\pi$ and round to the nearest tenth.

Know — You can use the area of a circle to find its radius using the formula $A = \pi r^2$.

Know — You can find the circumference of a circle using the formula $C = \pi d$.

Know — The value of $\pi$ is approximately 3.14; you can work with almost the exact value of $\pi$ by using the calculator.

Lesson 9: Area of a Sector Date: _____

## Standard(s)

◆ **G.11(B)** determine the area of ~~composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or~~ sectors of circles to solve problems using appropriate units of measure

## Important Vocabulary

- **sector**

## Notes for Intellectual Preparation & Lesson Planning

**Necessary Materials and Pre-Lesson Prep**

- Unit 8.9 Student Workbook pages
- Class set of red pens

Lesson Structure:

- Do Now (7 min)
- INM (15 min)
- Debrief (8 min)
- Student Practice (15 min)
- Exit Ticket (10 min)

**Mathematical Goal of this Lesson**

By the end of this lesson, students should be able to find the area of a sector by applying the knowledge that the area of a sector is proportional to the central angle.

**Opportunities to CFU**

✓ INM: 1, 2, 3, 6 ✓ Student Practice: 1, 2, 3, 4

**Other Notes to Inform Your Planning**

For the **Do Now:** This Do Now sets up the INM and cannot be skipped or replaced. It is important to ensure students know if they're correct before moving onto the INM; please ensure they understand why the correct answers are correct.

On **scaffolding**: For each question, coach struggling students to sketch the slices of pizza described just as they're directed to in #3. For students that struggle with division (e.g. $360 \div 12$) encourage use of the calculator; basic computations should not interfere with the developing an understanding of how to find the area of a sector.

**Focus on Disciplinary Literacy**

DEBRIEF

## Lesson Look Fors

Look for teachers to…

- ❑ emphasize that sector of a circle is a fraction of the pizza; if you know the area of the circle, you can find the area of the sector by setting up a fraction with the central angle over the total (e.g. $\frac{20°}{360°}$)

Look for students to…

- ❑ recall that multiplying by a unit fraction is the same as dividing by the denominator (e.g. multiplying by $\frac{1}{2}$ is the same as dividing by 2).
- ❑ see central angle measures as fractions with a denominator of 360°.

## Student Know/Do Chart

Do: Students can find the area of a sector of a circle given a radius and central angle measure.

Know: A sector is a fraction of the whole circle. If you have the central angle measure, you can make it the numerator in a fraction with a denominator of 360 and multiply it by the area of the circle to find the area of just the sector. In other words, $\frac{angle}{360} * \pi r^2 =$ area of sector.

Lesson 10: Arc Length | Date: _____

## Standard(s)

- **G.12(B)** apply the proportional relationship between the measure of an arc length of a circle and the circumference of the circle to solve problems

## Important Vocabulary

- **arc length**
- circumference

## Notes for Intellectual Preparation & Lesson Planning

**Necessary Materials and Pre-Lesson Prep**

- Unit 8.10 Student Workbook pages
- Class set of red pens, rulers, string, and protractors

Lesson Structure:

- Do Now (7 min)
- INM (15 min)
- Debrief (5 min)
- Student Practice (18 min)
- Exit Ticket (10 min)

**Mathematical Goal of this Lesson**

By the end of this lesson, students should be able to find a circle's requested arc length. Note that this is different from arc measure – an arc measure is given in degrees and is the same as the arc's corresponding central angle measure. Arc length is a part of a circle's circumference.

**Opportunities to CFU**

- ✓ INM: 3, 4, 6, 8
- ✓ Student Practice: 1, 2, 3, 4

**Other Notes to Inform Your Planning**

For the **Do Now:** This Do Now sets up the INM and cannot be skipped or replaced. Responses will vary, and that is okay. The goal of the Do Now is for students to identify and highlight the arc. Before moving on to the INM, ensure students have compared their work to an exemplar.

**About the Workbook page**: the workbooks are in black and white, so depending on the print quality, the map may not be easy to read. Consider projecting the color version as large as you can throughout the INM.

**Focus on Disciplinary Literacy**

DEBRIEF

For the **INM**: Note that students answers will vary a bit since they may identify different centers and make slightly different measurements for the radius and circumference. The goal is not that students all get the exact same answer, but that they understand the process of determining what fraction of the whole they have – to see that arc length and central angle are proportional.

## Lesson Look Fors

Look for teachers to…

- ❑ circulate and monitor, listening in on discussions and preselecting students to share what they notice.
- ❑ When circulating, ask the questions provided in the green box on TE p85 to support scaffolding

Look for students to…

- ❑ use string to measure arc length, similar to how they've been measuring circles in previous lesson.
- ❑ build on what they learned in the previous lesson and apply the same proportional reasoning to circumference instead of area.

## Student Know/Do Chart

Do: Students can find a requested arc length given a radius and central angle measure.

Know: An arc length is a fraction of a circle's entire circumference. If you have the central angle measure, you can make it the numerator in a fraction with a denominator of 360 and multiply it by the circumference of the circle to find the desired arc length. In other words, $\frac{angle}{360} * 2\pi r =$ requested arc length.

**Cumulative Review** | Success Day Date: _____

| Standard(s) | Notes for Intellectual Preparation & Lesson Planning | Lesson Look Fors |
|---|---|---|
| ◆ **G.11(B)** determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure<br>◆ **G.12(A)** apply theorems about circles, including relationships among angles, radii, chords, tangents, and secants, to solve non-contextual problems<br>◆ **G.12(B)** apply the proportional relationship between the measure of an arc length of a circle and the circumference of the circle to solve problems<br>◆ **G.12(E)** show that the equation of a circle with center at the origin and radius is $x^2 + y^2 = r^2$ and determine the equation for a graph of a circle with radius $r$ and center $(h, k)$, $(x - h)^2 + (y - k)^2 = r^2$. | **Necessary Materials and Pre-Lesson Prep**<br>▪ Review students' Unit 8 exit ticket data to determine what to prioritize during review<br>▪ Internalize Review Lesson 8.11 if you choose to use it<br><br>Lesson Structure:<br>Do Now (7 min)<br>INM (38 min)<br>Debrief (10 min)<br>Student Practice (0 min)<br>Exit Ticket (0 min)<br> <br><br>**Mathematical Goal of this Lesson**<br>By the end of this class period, students should get the opportunity to review major concepts from Unit 8.<br><br>**Other Notes to Inform Your Planning**<br>You should use this Success Day to review however you see fit. An optional review activity has been provided in the Teacher Edition called "Circuit Training" (Review Lesson 8.11). To prepare for today's Circuit Training review activity, ensure students have their Student Workbook. Plan to have extra copies of the Student Workbook Lesson 8.11 pages for students that do not have a workbook.<br><br>To complete this activity, students should start in the upper left box, work the problem, then search for the answer in one of the other boxes. Once they find it, they will label that box #2, then work the new problem. Students will continue in this manner until they reach the 16th question, whose answer should be in box 1. If students "short-circuit" and return to the first box without having completed all the problems, they will know there was a mistake made somewhere. Students may work on the circuit individually, though they generally prefer to work in pairs or groups of three. You may wish to provide whiteboards (or other non-permanent surfaces) for students to work out problems and as a tool to facilitate collaboration among group members.<br><br>Answers in the circuit are intentionally similar which keeps students from simply picking answers<br>from context clues alone. Common wrong answers are often the correct answers to other problems, so students have to attend to precision in their problem solving and be ready to go back to problems and revise their thinking. | Look for teachers to…<br>❑ facilitate a review session that prioritizes what students need based on previous exit ticket and/or quiz data.<br><br>Look for students to…<br>❑ review Unit 8 topics in preparation for the Unit 8 exam.<br><br>Student Know/Do Chart<br><br>Students can prepare for the Unit 8 Exam by reviewing Unit 8 content. |

## Recommended Unit 8 Success Day Material and Resources

Date: _____

For more practice with equations of circles (Lesson 1), try…

- 22-23 MCR Lesson 8.11: Equations of Circles: TE | SW
- Imagine Math: Equations of Circles: TE | SW
- ThatQuiz: Equations of Circles
- ThatQuiz: Equations of Circles 2
- Desmos: Equation of a Circle

For more practice with circle vocabulary (Lesson 2), try…

- 22-23 MCR Lesson 8.1: Introduction to Circles: SW | TE
- Kuta Software: Arcs and Central Angles
- Khan Academy: Arc Length practice

For more practice with properties of tangents (Lesson 3), try…

- 22-23 MCR Lesson 8.6: Tangent Lines: SW | TE
- Geometry Coach: Tangent Lines
- GeoGebra: Properties of Tangents Drawn to Circles (B)
- GeoGebra: Tangent Chord Angles

For more practice with properties of chords and arcs (Lessons 4-5), try…

- 22-23 MCR Lesson 8.2: Central Angles and Arc Measures: SW | TE
- 22-23 MCR Lesson 8.4: Chords and Arcs: SW | TE
- Calc Workshop: Arc of a Circle
- GeoGebra: Perpendicular Bisector of a Chord
- GeoGebra: Properties of Chords in a Circle

For more practice with inscribed angles (Lesson 6), try…

- 22-23 MCR Lesson 8.5: Inscribed Angles: SW | TE
- Wolfram Demonstrations Project: Inscribed and Central Angles in a Circle

For more practice with area and circumference of a circle and/or area of a sector and arc lengths (Lessons 8-10), try…

- 22-23 MCR Lesson 8.3: Circumference and Arc Length: SW | TE

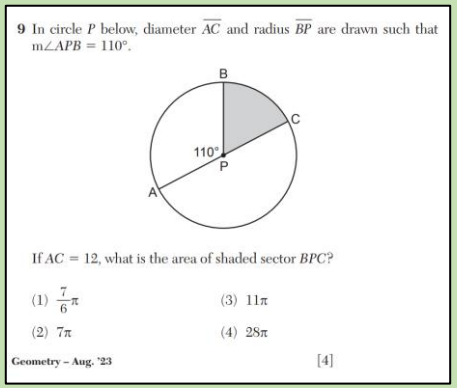
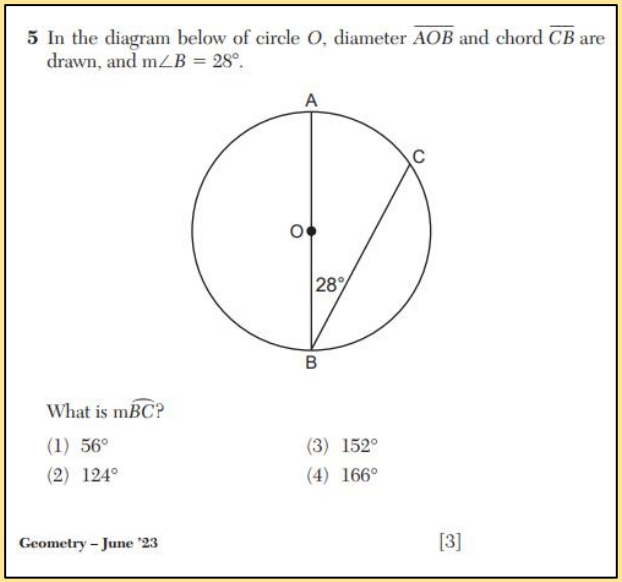
For a combination of problem sets on various circle concepts, try…

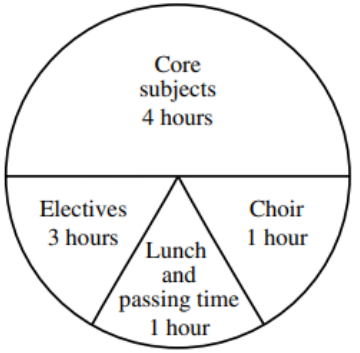
- Progressive Math Initiative: Unit 8 Circles Packet

| Unit 8 Exam | Date: _____ |
|---|---|
| Standard(s) | Notes for Intellectual Preparation & Lesson Planning |
| ◆ **G.11(B)** determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure<br><br>◆ **G.12(A)** apply theorems about circles, including relationships among angles, radii, chords, tangents, and secants, to solve non-contextual problems<br><br>◆ **G.12(B)** apply the proportional relationship between the measure of an arc length of a circle and the circumference of the circle to solve problems<br><br>◆ **G.12(E)** show that the equation of a circle with center at the origin and radius is $x^2 + y^2 = r^2$ and determine the equation for a graph of a circle with radius $r$ and center $(h, k), (x - h)^2 + (y - k)^2 = r^2$. | **Necessary Materials and Pre-Lesson Prep**<br>▪ Print enough copies of UE8 Bluebook on Curriculum Corner (for the FRQ) and ensure you have access to the exam on Edcite. (If you do not have access to the exam, notify your TC and submit a Zendesk ticket ASAP.)<br><br>**Notes to Inform Your Planning**<br>Review the Unit 8 Exam on Curriculum Corner. Internalize and create an exemplar for the assessment prior to teaching the unit as part of unpacking the unit. Use your exemplar to spar with the solutions provided in the Assessment Companion on Curriculum Corner.<br><br>The scanning deadline for the Unit 8 Exam is **March 5th, 2026**. Consider administering the exam 1-3 school days BEFORE March 5th to allow sufficient time for grading the FRQ.<br><br>Refer to the scoring guide to score the FRQ. |

## UNPACKED STANDARDS

Focus standards for this unit.

| Standards Clarification | | |
|---|---|---|
| **Standards** | **Specificity** | **Notes/Explanations/Examples** |
| **G.11B** determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure | Content:<br>• area of circles<br>• area of sectors<br><br>Including:<br>• finding the area of sectors and circles<br><br>Excluding:<br>• figures that are not circles | **Geometry Regents Exam: August 2023**<br>  <br>9 In circle $P$ below, diameter $\overline{AC}$ and radius $\overline{BP}$ are drawn such that $m\angle APB = 110°$.<br>B<br>C<br>110°<br>P<br>A<br>If $AC = 12$, what is the area of shaded sector $BPC$?<br>(1) $\frac{7}{6}\pi$ (3) $11\pi$<br>(2) $7\pi$ (4) $28\pi$<br>Geometry – Aug. '23 [4] |
| **G.12A** apply theorems about circles, including relationships among angles, radii, chords, tangents, and secants, to solve non-contextual problems | Content:<br>• Radius<br>• Diameter<br>• Circumference<br>• pi<br>• Chord<br>• Central angles<br>• Arc measures<br>• Major, major, and adjacent arcs<br>• Semicircles<br>• Inscribed angles<br>• Intercepted arc<br>• Tangent<br>• Point of tangency<br>• Common tangent<br>• Chord segments<br>• Tangent segments<br><br>Including but not limited to:<br>• Identifying parts of a circle<br>• Identifying central angles, major arcs, and semicircles, and finding their measures<br>• Using relationships between arcs and chords to find missing values<br>• Finding measures of inscribed angles<br>• Solving problems involving circumscribed polygons<br>• Finding measures of segments that intersect in the interior or exterior of a circle | **Geometry Regents Exam: June 2023**<br>  <br>5 In the diagram below of circle $O$, diameter $\overline{AOB}$ and chord $\overline{CB}$ are drawn, and $m\angle B = 28°$.<br>A<br>C<br>O<br>28°<br>B<br>What is $m\overset{\frown}{BC}$?<br>(1) 56° (3) 152°<br>(2) 124° (4) 166°<br>Geometry – June '23 [3] |

| Standards Clarification | | |
|---|---|---|
| **Standards** | **Specificity** | **Notes/Explanations/Examples** |
| **G.12B** apply the proportional relationship between the measure of an arc length of a circle and the circumference of the circle to solve problems | Content:<br>• Arc length<br>• Radian measure<br>• Inscribed polygons<br>• Circles circumscribed about polygons<br><br>Including but not limited to:<br>• Setting up and solving proportions to give arc lengths, degree measures, and/or circumference | **ACT: 2014 – 2015 release**<br><br>**14.** Antwan drew the circle graph below describing his time spent at school in 1 day. His teacher said that the numbers of hours listed were correct, but that the central angle measures for the sectors were not correct. What should be the central angle measure for the Core subjects sector?<br><br>**F.** 72°<br>**G.** 80°<br>**H.** 160°<br>**J.** 200°<br>**K.** 288°<br><br>  <br> |
| **G.12E** show that the equation of a circle with center at the origin and radius $r$ is $x^2 + y^2 = r^2$ and determine the equation for the graph of a circle with radius $r$ and center $(h, k)$, $(x - h)^2 + (y - k)^2 = r^2$. | Content:<br>• Equation of a circle<br>• Pythagorean Theorem<br>• Distance Formula<br><br>Including but not limited to:<br>• Graphing a circle when given an equation<br>• Finding the equation of a circle when given a circle on a coordinate plane | **ACT: 2021 – 2022 release**<br><br>**13** The equation of a circle is $x^2 + y^2 + 12x = -27$. What are the coordinates of the center and the length of the radius of the circle?<br><br>(1) center $(6,0)$ and radius 3<br>(2) center $(6,0)$ and radius 9<br>(3) center $(-6,0)$ and radius 3<br>(4) center $(-6,0)$ and radius 9 |

## VERTICAL STANDARDS

This section details the **progression** of key student expectations/standards** in the courses **before** and **after** this course. This will help you understand what **prior knowledge skills to build upon** and guide you in knowing what **skills you are preparing your students** for in the subsequent course.

### 7th Grade

**7.4E** convert between measurement systems, including the use of proportions and the use of unit rates.

**7.5B** describe π as the ratio of the circumference of a circle to its diameter.

**7.8C** use models to determine the approximate formulas for the circumference and area of a circle and connect the models to the actual formulas.

**7.9B** determine the circumference and area of circles.

### Algebra I

**A.6B** write equations of quadratic functions given the vertex and another point on the graph, write the equation in vertex form ($f(x) = a(x - h)^2 + k$), and rewrite the equation from vertex form to standard form ($f(x) = ax^2 + bx + c$).

**A.6C** write quadratic functions when given real solutions and graphs of their related equations.

**A.7C** determine the effects on the graph of the parent function $f(x) = x^2$ when f(x) is replaced by af(x), f(x) + d, f(x - c), f(bx) for specific values of a, b, c, and d.

### Geometry

**G.12A** apply theorems about circles, including relationships among angles, radii, chords, tangents, and secants, to solve non-contextual problems.

**G.12B** apply the proportional relationship between the measure of an arc length of a circle and the circumference of the circle to solve problems.

**G.12E** show that the equation of a circle with center at the origin and radius $r$ is $x^2 + y^2 = r^2$ and determine the equation for the graph of a circle with radius $r$ and center $(h, k)$, $(x - h)^2 + (y - k)^2 = r^2$.

**G.11B** determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure

### Algebra II

**2A.4B** write the equation of a parabola using given attributes, including vertex, focus, directrix, axis of symmetry, and direction of opening.

**2A.4D** transform a quadratic function $f(x) = ax^2 + bx + c$ to the form $f(x) = a(x - h)^2 + k$ to identify the different attributes of f(x).

**2A.6A** analyze the effect on the graphs of $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$ when f(x) is replaced by af(x), f(bx), f(x - c), and f(x) + d for specific positive and negative real values of a, b, c, and d.

**2A.6C** analyze the effect on the graphs of $f(x) = |x|$ when f(x) is replaced by af(x), f(bx), f(x- c), and f(x) + d for specific positive and negative real values of a, b, c, and d.

**2A.6G** analyze the effect on the graphs of $f(x) = 1/x$ when f(x) is replaced by af(x), f(bx), f(x- c), and f(x) + d for specific positive & negative real values of a, b, c, & d.

### Pre-Calculus

**P.3F** determine the conic section formed when a plane intersects a double-napped cone.

**P.3G** make connections between the locus definition of conic sections and their equations in rectangular coordinates.

**P.4C** represent angles in radians or degrees based on the concept of rotation and find the measure of reference angles and angles in standard position.